

STATISTICAL TECHNIQUES IN LOW FLOW HYDROLOGY

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ABSTRACT

Low flow hydrology is used in water resources problems like water-quality management applications, water supply planning and determination of minimum downstream release requirements from hydropower, water supply, cooling plants and other facilities. In examining low flows, different duration-days (D-day) of flows are taken into consideration. In USA, 7-day 10-year low flow is accepted as low flow criteria. The Weibull distribution accepted as the probability distribution of low flows by many researchers. On arid and cool regions, hydrologists may encounter data series that contain zero values. These zero flows have to be taken into account in estimating the probabilities of low flows by using the theorem of total probability.

Introduction

Low flow discharges occur at some period of the year and sometimes get dry, especially in arid and cold regions. This usually occurs during the summer when irrigation is of primary importance. From the dilution point of view, this has important consequences for the discharge of wastewater into the flows during the low flow period. If the flow decreases below a certain value, there is direct effect on the aquatic life of the river flow under consideration. Low flow statistics are also used in water planning to determine allowable water transfers and withdrawals. Other applications of low flow frequency analysis include determination of minimum downstream release requirements from hydropower, water supply, cooling plants and other facilities.

In examining low flows, different duration-days (D-day) of flows are taken into consideration. For instance, in calculating 3-day of N year of observations, mean value of the three days of succeeding flows are calculated and exceedance probabilities are determined. By the help of flow duration curve of D-day flow, one can find out the percentage of time during which specified discharges are equalled or exceeded during the period of record.

Also probability distribution functions are used in low flow analysis. While the flow duration curve is concerned with the proportion of time during which a flow exceeded, or equivalently the average interval in years that the river flow falls below a given discharge. In USA 7-day 10-year low flow ($Q_{7,10}$) value is accepted as a low flow criteria. Some researchers accept 7-day 2-year flow ($Q_{7,2}$) as low flow criteria. (Task Committee, 1980).

Probability Distribution of Low Flows

For predicting $Q_{7,10}$ discharge, different distribution functions and parameter estimation techniques have been investigated (Gumbel, 1954, 1958; Matalas, 1963; Condie & Nix, 1975). In a study using Lognormal (LN2), Weibull (W2) and Gumbel distribution performed by Vogel and Kroll (1989) in USA, they came to the conclusion that LN2 distribution is the best fitted distribution for estimating $Q_{7,10}$ discharge at a station and on a regional basis. In an another study, Bulu and Onoz (1997) found out that W2 distribution conformed better to the low flows of the selected rivers in Turkey. In the FRIEND study group, the Weibull distribution has been adopted for low flow analysis (Gustard et al., 1989) In this study, W2 distribution and its parameter estimation techniques will be given.

The probability density function of W2 distribution has the form of,

$$f(x) = \alpha x^{\alpha-1} b^{-\alpha} \exp\left[-\left(\frac{x}{b}\right)^\alpha\right] \quad x \geq 0; \alpha, \beta > 0 \quad (1)$$

Where α is the scale parameter and β is the location parameter. Thus the cumulative frequency function is,

$$F(x) = 1 - \exp\left[-\left(\frac{x}{b}\right)^\alpha\right] \quad (2)$$

The mean, variance and the skewness coefficient have been given respectively as,

$$E(x) = b \Gamma\left(1 + \frac{1}{\alpha}\right) \quad (3)$$

$$\text{var}(x) = b^2 \left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \Gamma^2\left(1 + \frac{1}{\alpha}\right) \right] \quad (4)$$

$$g = \frac{\Gamma\left(1 + \frac{3}{\alpha}\right) - 3\Gamma\left(1 + \frac{2}{\alpha}\right)\Gamma\left(1 + \frac{1}{\alpha}\right) + 2\Gamma^3\left(1 + \frac{1}{\alpha}\right)}{\left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \Gamma^2\left(1 + \frac{1}{\alpha}\right) \right]^{2/3}} \quad (5)$$

Parameter Estimation by Method of Moments (MOM)

The parameters of the W2 distribution can be estimated by the MOM by substituting the mean of and variance respectively in Equations (3) and (4) and then solving the two equations simultaneously for α and β .

Parameter Estimation by Maximum Likelihood Method (MLE)

The estimation of α and β parameters by MLE has been given by Haan (1977). These estimates can be determined by letting,

$$I = b^{-a} \quad (6)$$

And then solving

$$I = \frac{n}{\sum_{i=1}^n x_i^a} \quad (7)$$

And

$$a = \frac{n}{I \sum_{i=1}^n x_i^a \ln x_i - \sum_{i=1}^n \ln x_i} \quad (8)$$

Simultaneously for α . β is calculated by,

$$b = (I)^{-\frac{1}{a}} \quad (9)$$

Parameter Estimation by Probability Weighted Moments (PWM)

L-moments and PWM methods were preferred for the parameter estimation of the probability distribution functions since this technique gives more robust parameter estimates. As was mentioned in Stedinger et al. (1993), L-moments also provide simple and reasonably efficient estimators of the characteristics of hydrologic data and of a distribution's parameters.

The estimate of α and β parameters by PWM are,

$$a = \frac{\ln 2}{L_{2,(\ln x)}} \quad (10)$$

$$b = \exp\left(L_{1,(L_{nx})} + \frac{0.5772}{a}\right) \quad (11)$$

Once α and β are estimated by any of the methods given above, low flow discharge for a taken T return period can be calculated by,

$$x = b \left[-Ln\left(1 - \frac{1}{T}\right) \right]^{1/a} \quad (12)$$

As given by Kite (1974).

Probability Estimation of Zero Flows

On semi-arid and arid regions, hydrologists may encounter data series that contain zero values. Such data sets are called *censored samples* (Stedinger et al., 1993)

These zero flows have to be taken into consideration in estimating the probabilities of low flows (Haan, 1997). According to the total probability theorem,

$$P(X \geq x) = P(X \geq x | X = 0)P(X = 0) + P(X \geq x | X \neq 0)P(X \neq 0) \quad (13)$$

Since $P(X \geq x | X = 0)$ is zero, the relationship reduces to,

$$P(X \geq x) = P(X \geq x | X \neq 0)P(X \neq 0) \quad (14)$$

In this relationship $P(X \neq 0)$ would be estimated by the fraction of non-zero values, k , and $P(X \geq x | X \neq 0)$ would be estimated by a standard analysis of the non-zero values with the same size taken to be equal to the number of non-zero values. This relation can be written as a function of cumulative probability distributions,

$$1 - F(x) = k[1 - F^*(x)] \quad (15)$$

Or

$$F(x) = 1 - k + kF^*(x) \quad (16)$$

Where $F(x)$ is the cumulative probability distribution of all X $[P(X \leq x | X \geq 0)]$, k is the probability that X is not zero, and $F^*(x)$ is the cumulative probability distribution of the non-zero values of X , $[P(X \leq x | X \neq 0)]$. This is mixed distribution which has a finite probability for $X=0$ and a continuous probability distribution for $X>0$ (Bulu, 1997).

Equ. (16) can be used to estimate a magnitude of an event with a return period T by solving first for $F^*(x)$ and then using the inverse transformation of $F^*(x)$ to get the value of X . This merely depends on the probability distribution function applied to the non-zero flow values.

$$F^*(x) = \frac{F(x) - 1 + k}{k} \quad (17)$$

Since the return period of the flow can be estimated by,

$$T = \frac{1}{F(x)} \quad (18)$$

Equ. (17) takes the form of,

$$F^*(x) = \frac{\frac{1}{T} - 1 + k}{k} \quad (19)$$

The applicability of Equ. (19) Depends upon to get positive values of probabilities, $F^*(x)$. So that, the application of total probability theorem to the low flows depends on the relations between T and k . For different return periods, k fractions should be,

$$k \geq \frac{T-1}{T} \quad (20)$$

For the application of this theorem. Actually, if we obtain negative $F^*(x)$ values by Equ. (19), it means that for that return period T and fraction k , the probability of seeing that flow is zero for the river under consideration.

Case Study

Twenty-three years of 7-day low flow data of Hayrabolu River in the Thrace region of Turkey are available. Flow duration curve of 23 years of 7-day flows was given in Figure 1. Q_{90} , Q_{95} and Q_{99} low flow discharges obtained from the duration curve were also given in Figure 1.

Statistical analysis was performed to this data. Five of the values are zero and the remaining 18 values have a mean of $0.11 \text{ m}^3/\text{sec}$, a standard deviation of $0.08 \text{ m}^3/\text{sec}$, and are distributed as 2-parameter Weibull distribution with parameters $\alpha=1.273$, $\beta=0.120$ estimated by MLE method. The k parameter is estimated as $18/23=0.783$ to calculate the return period of zero flows. Since $F^*(0)=0$, by using Equ. (16), $F(0)=0.217$ and from Equ. (18), $T=4.16$ years was found. This value shows that the probability of seeing 7-day low flows greater 4.16 years of return period is zero.

The 7day low flow value with $T=4$ years return period can be obtained by the help of Equ. (19) As

$$F^*(x) = \frac{\frac{1}{4} - 1 + 0.783}{0.783} = 0.042$$

Low flow discharge for $F^*(x)=0.042$ can be calculated from the 2-parameter Weibull distribution by the help of of Equ. (12) As

$$x = 0.12[-\ln(1 - 0.042)]^{1/1.273} = 0.01 \text{ m}^3 / \text{sec}$$

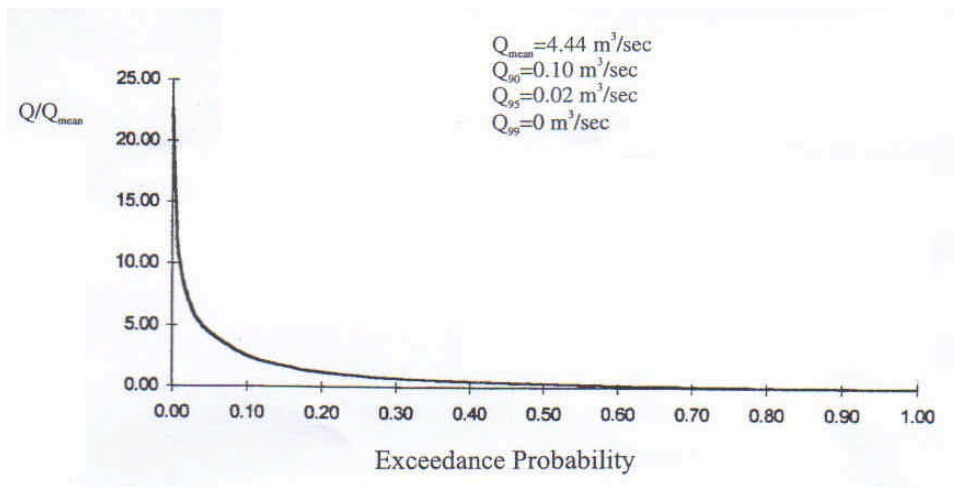


Figure 1. Flow duration curve 7-day low flow values of the Hayrabolu River

Conclusions

The following conclusions can be presented for this study:

1. Low flow hydrology is used in water quality management applications and water supply planning to determine allowable water transfer and withdrawals.
2. The 2-parameter Weibull distribution is the generally accepted and applied distribution function for the frequency analysis of low flows.
3. Total probability theorem can be used for the frequency analysis of low flows when the hydrologic data series contain zero values, which is the case for arid regions.

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